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50. Internationales Wissenschaftliches Kolloquium
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Investigating the Ability of Holonomic Automatic Systems' Dynamics Control

ABSTRACT

The aim of investigation is the building of differential analyzer for reproduction of curves lying on quadric surfaces. The profile of affix speed is to be set. The computer simulation of the worked out mathematical model of differential analyzer and investigation of the characteristics of affix movement are carried out using MATLAB/SIMULINK. When speaking about holonomic automatic systems' dynamics control, it's useful to mention the ability of acceleration control. The problem of acceleration control is very important, as acceleration and velocity of affix movement define the productivity of manufacturing equipment. A set of used approaches of acceleration control leads to very huge and 'hard-to-calculate' models and algorithm. A new approach of acceleration control based of velocity profile assignment is proposed in the paper. The advantages of this control algorithm are simplicity of model, quick and efficient algorithm of control. The argument control algorithm is used for both velocity and acceleration control. This approach excludes the redundancy of classical based models proposed for velocity and acceleration control, when two different simulation structures are alternately used to control acceleration and velocity.

1. HOLONOMIC AUTOMATIC SYSTEMS' DEFINITION

Any process correct passing can be characterized by fulfillment of some definite conditions, which leads to goal achievement. For holonomic automatic systems these conditions can be written in the following way:

$$F_j(t, x_1, x_2, ..., x_n) = 0; \quad j = 1,2,...m \tag{1}$$

between n variables of the system of equations, where F_j are differentiable in established range of variables.

The system can be described by system of differential equations. The first problem is: which is the structure of differential equations of the system. Having found the structure of differential equations' system, one can easily either change the differential equations of the system itself, so those would match to the found ones or use special control signals.

As an example of holonomic automatic system we can consider a two-coordinate milling machine, which can treat different profiles. For example, the task for that milling machine can be as follows: reproduce the quadric curve:

$$Ax^2 + 2Bxy + Cy^2 + 2Dx + 2Ey + F = 0 \quad (2)$$

beginning from point $x_0 y_0$ and ending in point $x_1 y_1$. Having processed that curve, a new task can be given to a milling machine and so on. So, to carry out that task, the inputs of the system must be fed with some definite control signals. Software controlled systems greatly decrease the amount of information which is passed from a human to a machine because an object in that case is controlled with the help of mathematical model and all control information is the model reorganization, but not the building of a new control model. It's obvious that in case of control information flow decrease its meaning content remains the same.

The decrease of information flow from a human to a machine is one of the main traits of automation process.

A large quantity of modern technical systems, especially mechanotronic, is based on trajectory constructing on topological manifolds (particularly on curves and surfaces) with established accuracy. For automated control of these systems it is preferable for travel task to be given in the form of finite equation or system of equations which connect single coordinates. These systems with the behavior which is established with exactness up to variety intersection are related to the class of holonomic systems.

The problem of synthesis of holonomic systems presupposes the synthesis of differential analyzer in the form of differential equations for which the solution is the reproducible program of movement. The building of multi-coordinate control systems based on differential analyzers gives such advantages as:

1. Simplification of control algorithm and automatic control system structure;
2. Possibility to control the speed of affix movement without considerable complication of structure of automatic control system;
3. Possibility of change-over of control system parameters for reproduction of affix movement on different topological manifold without considerable complication of structure of automatic control system;
4. Possibility of optimal curves programming, i.e. such control design that provides maximum performance of manufacturing equipment while surface treatment;
5. Possibility of universal software development for machines control for surface treatment.

Portraying varieties in a general sense is very complex task, but presenting some of them in 3-D space can give necessary understanding of what they are. Below (Fig. 1) are some of them.

It's obvious that a sphere, for example, can have infinite number of curves lying on it, so, a sphere incorporates all possible curves belonging to it. So, a sphere, being a variety, can be described by some system of differential equation, while the solution of that system of equation is the curve which belongs to the sphere.

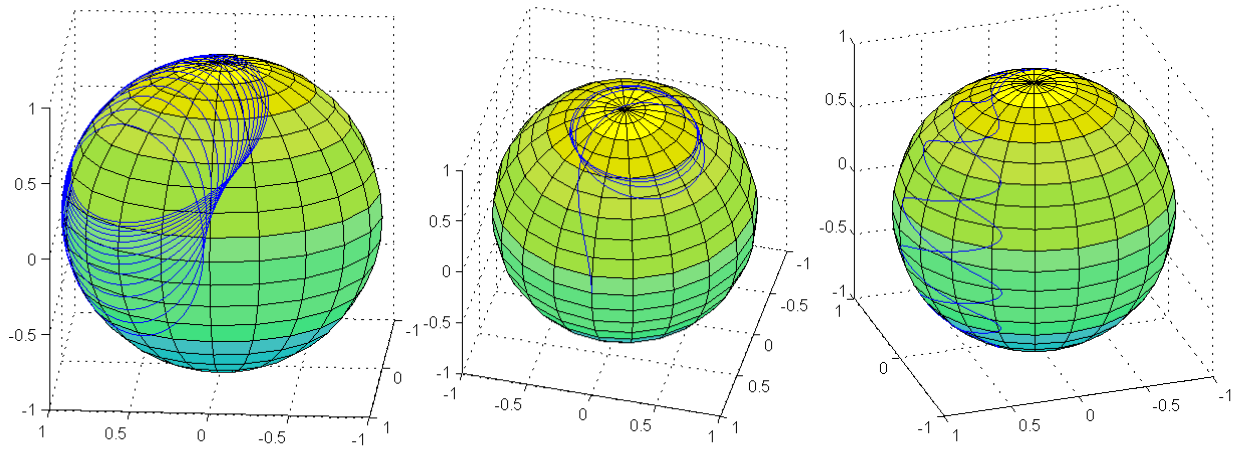

Fig.1. Examples of holonomic automatic systems

2. DIFFERENTIAL EQUATIONS' STRUCTURE

Differential analyzers synthesis, which reproduce this or that function on a variety, comes to definition of system of differential equations, the salvation of which is the reproducible function. This task is ambiguous, and its rational solution depends on the approaches which help combine required differential system of equation and which function's properties were used when finding that system of equations. One of the methods of differential analyzers' synthesis is parametric method of differential analyzers synthesis.

3. PARAMETRIC METHOD OF DIFFERENTIAL ANALYZERS SYNTHESIS

Let us examine the parametric method of differential analyzers synthesis. Any multivariable function can be presented in the form of finite superposition of continuous functions of one-variable functions. So, the problem of differential analyzer synthesis for a definite function reproduction can be reduced to the problem of differential analyzer synthesis for reproduction of one-variable function.

Suppose an implicit function of *n* variables:

$$F(x_1, x_2, ..., x_n) = 0\,, \tag{3}$$

which is differentiable in established range of variables *M* and is not a hypertranscendental function. We shall look for the first-order system of differential equations:

$$\frac{dx_i}{d\varphi} = f_i, i = 1,2,...,n, \tag{4}$$

the solution of which satisfies the equation (3) in established range of variables *M*.

The variable φ in (4) is an argument of the designed differential analyzer. In is necessary to define the functions f_i. After differentiation of equation (3) by parameter φ, we shall get:

$$\sum_{i=1}^{n} \frac{\partial F}{\partial x_i} \frac{dx_i}{d\varphi} = 0 \tag{5}$$

If the system (4) solution turns into identity the equation (3), then system (4) turns into identity the equation (5). Thus, the functions f_i definition can be based on analytical condition (5). This problem has a solution set, at that functions f_i in all cases depend on partial derivatives $\frac{dF}{dx_i}$. For analytical algorithm simplification let us concern the functions f_i are linear functions of mentioned above partial derivatives. This method of differential analyzers synthesis has an essential advantage: the argument φ, which is concerned to be a system parameter, can be any analytical function, what specifically lets realize the argument control, which is necessary for differential analyzer structure simplification.

Let us consider the differential analyzer synthesis consecution, so the affix coordinates would belong to trajectories lying on the quadric surfaces.

The general equation of a quadric surface in a Cartesian space *Oxyz* is:

$$F(x,y,z) = a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{23}yz + 2a_{31}zx + 2a_{14}x + 2a_{24}y + 2a_{34}z + a_{44} = 0$$

The system of differential equations, describing a differential analyzer, for linear functions of second member, generally is:

$$\left.\begin{aligned} \frac{dx}{d\varphi} &= b_0 + b_1 x + b_2 y + b_3 z, \\ \frac{dy}{d\varphi} &= c_0 + c_1 x + c_2 y + c_3 z, \\ \frac{dz}{d\varphi} &= d_0 + d_1 x + d_2 y + d_3 z. \end{aligned}\right\}, \tag{6}$$

where b_i, c_i, d_i (i=0,1,2,3) are the coefficients of linear second member functions. The necessary condition of affix movement on a quadric surface is [1]:

$$\frac{\partial F}{\partial x}\frac{dx}{d\varphi} + \frac{\partial F}{\partial y}\frac{dy}{d\varphi} + \frac{\partial F}{\partial z}\frac{dz}{d\varphi} = 0. \tag{7}$$

Relying on equation (7), taking into account (5) and (6), we shall find the coefficients b_i, c_i, d_i (i=0,1,2,3) of the system of differential equations (6).

4. AN EXAMPLE OF DIFFERENTIAL ANALYZER SYNTHESIS FOR REPRODUCTION OF CURVES LYING ON A SPHERE

Let us examine an example of differential analyzer synthesis for reproduction of curves lying on a sphere $x^2+y^2+z^2=R^2$.

After differentiation of $\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}$ *and* $\frac{\partial F}{\partial z}$, basing on equations (6), (7), we shall find out the system of linear equations for coefficients b_i, c_i, d_i (i=0,1,2,3) definition [2]:

$$\left.\begin{aligned} & b_1 = c_2 = d_3 = b_0 = c_0 = d_0 = 0, \\ & b_2 + c_1 = 0, \\ & b_3 + d_1 = 0, \\ & c_3 + d_2 = 0. \end{aligned}\right\} \tag{8}$$

Assuming $b_2 = \frac{u_1}{b^2}$, $b_3 = \frac{-u_2}{c^2}$, $c_3 = \frac{u_3}{c^2}$, we shall find out that $c_1 = \frac{-u_1}{a^2}$, $d_1 = \frac{u_2}{a^2}$, $d_2 = \frac{-u_3}{b^2}$.

After substituting the values of b_i, c_i, d_i (i=0,1,2,3) into equation (6), we shall find the system of differential equations, describing the differential analyzer for programming of affix movement over a sphere.

$$\left.\begin{aligned} & \frac{dx}{d\varphi} = u_1 y - u_2 z, \\ & \frac{dy}{d\varphi} = -u_1 x + u_3 z, \\ & \frac{dz}{d\varphi} = u_2 x - u_3 y. \end{aligned}\right\} \tag{9}$$

where u_1, u_2, u_3 are control signals, which determine the form of a curve.

Assuming $d\varphi=dt$, the block-diagram of differential analyzer which produces the curves on a sphere, presented in MATLAB/Simulink, will be as presented on fig. 2.

For computer simulation of differential analyzer, the system MATLAB/Simulink was used. This software package features a graphical interface for constructing models in a block-diagram-like fashion, provides algorithms for numerical integration, and offers access to the MATLAB tools for displaying and analyzing the produced output.

Simulink has in fact the capability to describe the linear and non-linear systems without using differential equations at all. Like in a block diagram, the immediate relations between system variables, including nonlinearities, can be encoded and linked by data flow paths.

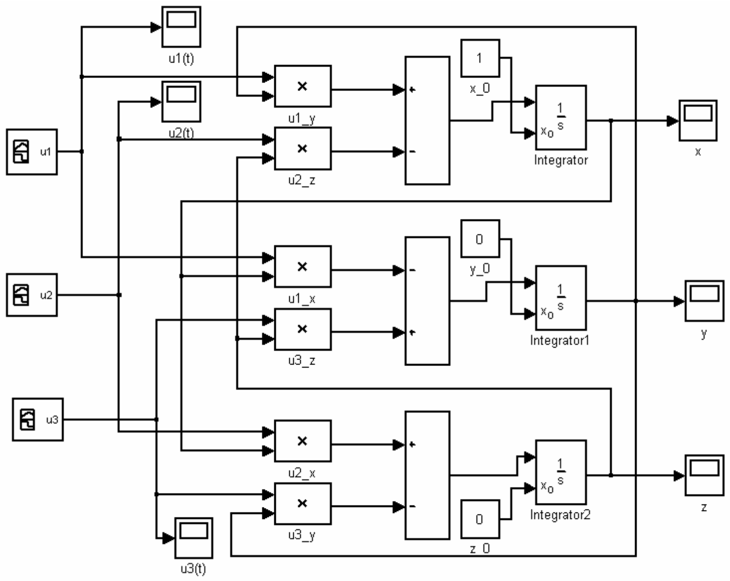

Fig.2. The structure of differential analyzer, reproducting the curves lying on a sphere

5. THE RESULTS OF COMPUTER SIMULATION OF THE DIFFERENTIAL ANALYZER IN MATLAB/SIMULINK

This subsection covers the results of computer simulation of a model, described above. As the input signals of the model, the values of initial conditions x_0, y_0, z_0 and the time-varying control signals were used. The outputs of the model are the coordinates of affix x, y, z. For differential analyzers affix movement parameters analysis, the comparison of two cases was carried out.

1. $x_0=1, y_0=0, z_0=0, u_1=5, u_2=5, u_3=5;$ time of affix movement is $t=10$ seconds. The trajectory is presented on Fig.3a;
2. $x_0=1, y_0=0, z_0=0, u_1=t, u_2=5, u_3=5, t=10$. The trajectory is presented on Fig3b.

a) b)

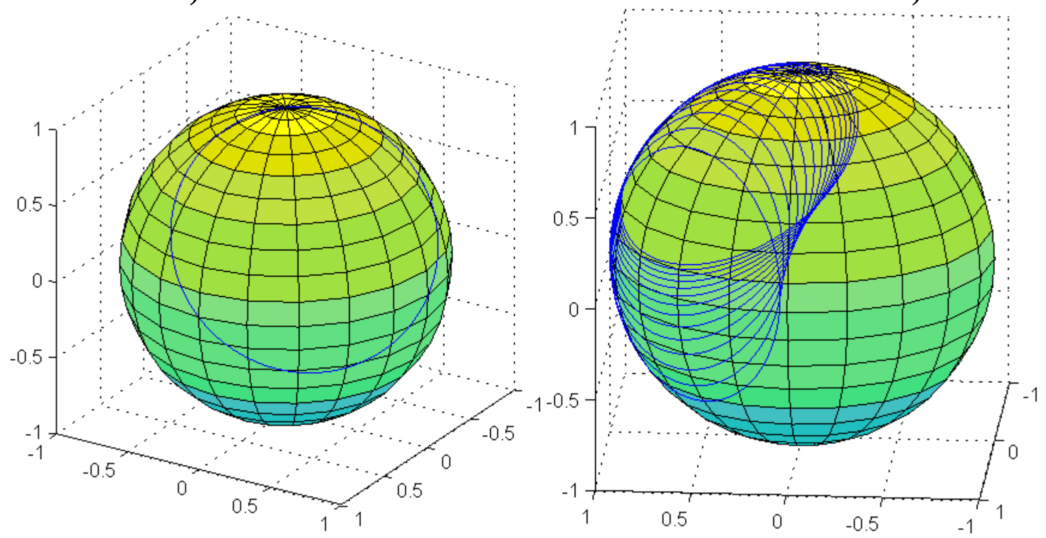

Fig. 3. Trajectories of affix movement

As it is seen, both trajectories lie on a sphere (a variety), however the trajectories themselves vary. It is significant that the type of a curve is determined by control signals u_1, u_2, u_3, whereas the surface itself is determined by the structure of differential analyzer.

What concerns the dynamic characteristics of affix movement, it is significant to mention the profile of affix velocity. For our case, the velocity profiles are presented on Fig.4. and Fig.5.

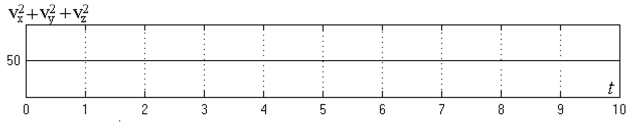

Fig.4. The affix velocity profile for $u_1=5$, $u_2=5$, $u_3=5$

As presented on Fig.4., one would make sure that the affix velocity is constant when all u_i (i=1,2,3) are constant. This is not applied when we have time-varying u_i.

The problem of velocity stabilization can be solved by argument control approach when concerning $d\varphi=\omega dt$, where ω is a time-varying function. It is significant that the argument control approach can be used to simplify the structure of differential analyzer.

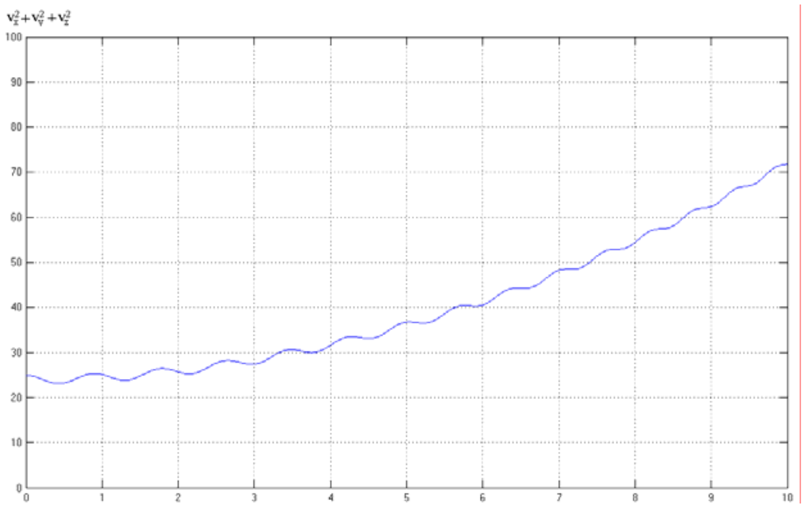

Fig.5. The affix velocity profile for $x_0=1$, $y_0=0$, $z_0=0$, $u_1=t$, $u_2=5$, $u_3=5$

6. HOLONOMIC AUTOMATIC SYSTEMS WITH ARGUMENT CORRECTION FOR VELOCITY STABILIZATION

The synthesis of differential analyzer for reproducing curves lying on a sphere is essential task; anyway implementation of mentioned above systems for an application presupposes the ability of dynamics control for such systems. Controlling at least velocity and profile is mandatory for technical systems. While a variety of methods of dynamics control exist, one of the most easy-to-use is argument control.

The argument of differential equations of holonomic systems can be represented by any function. The argument control can be used to form definite dynamic characteristics of outlet signals of programmer. The argument correction is used to form the definite profile of velocity of affix movement, at that the velocity value can be constant or variable.

For the purposes of velocity stabilization, the following system of equations must be solved:

$$\left.\begin{array}{l}\left.\begin{array}{l}F_1(x_1, x_2, ..., x_n) = 0 \\ F_2(x_1, x_2, ..., x_n) = 0 \\ \dots\dots\dots\dots\dots\dots \\ F_{n-1}(x_1, x_2, ..., x_n) = 0\end{array}\right\}(a) \\ \dot{x}_1^2 + \dot{x}_2^2 + ... + \dot{x}_n^2 = V^2 (b)\end{array}\right\} \tag{10}$$

Assuming $d\varphi = \omega dt$ in equation (6), and finding out $\frac{dx_i}{dt} = \omega f_i(x_1, x_2, ..., x_n)$, after substituting these into (10,b), we shall receive the following:

$$\omega^2 = \frac{V^2}{f_1^2(x_1, x_2, ..., x_n) + f_2^2(x_1, x_2, ..., x_n) + ... + f_n^2(x_1, x_2, ..., x_n)} \tag{11}$$

So, assuming that the argument of integrators, solving the equations (10), will be found as (11), the affix movement programming will be realized with pre-defined velocity *V*.

7. COMPUTER SIMULATION OF DIFFERENTIAL ANALYZER WITH DEFINITE VELOCITY PROFILE

The subsection covers the differential analyzer synthesis procedure for curves reproduction lying on a sphere with a constant velocity value $v_x^2 + v_y^2 + v_z^2 = V^2$.

Referring to (6) and assuming $d\varphi = \omega dt$, we shall obtain the following equations for ω^2:

$$\omega^2 = \frac{V^2}{(u_1 y - u_2 z)^2 + (-u_1 x + u_3 z)^2 + (u_2 x - u_3 y)^2} \tag{12}$$

Hence we get ω:

$$\omega = \pm \frac{V}{\sqrt{(u_1 y - u_2 z)^2 + (-u_1 x + u_3 z)^2 + (u_2 x - u_3 y)^2}} \tag{13}$$

One should remember that the sign of ω determines the direction of movement along the curve.

The structure of differential analyzer is presented on Fig.6. The results of computer simulation are presented on Fig.7.

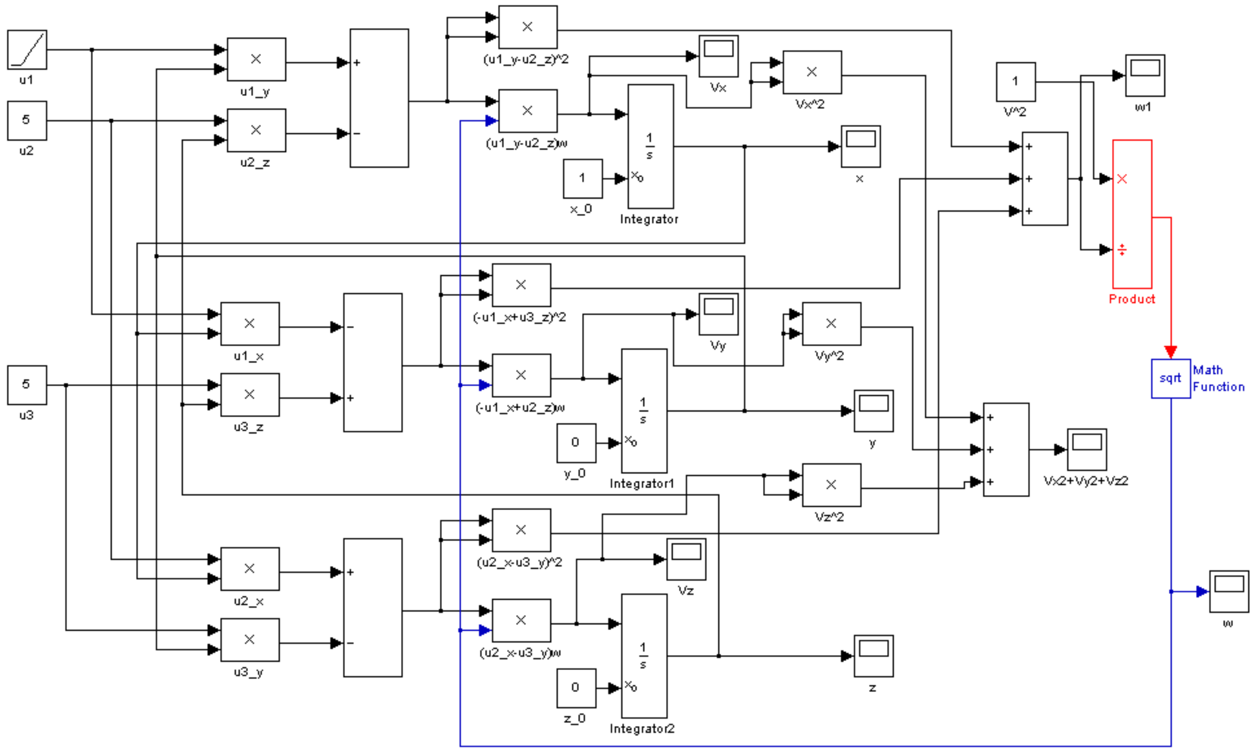

Fig.6. The block diagram of differential analyzer with stabilized affix velocity

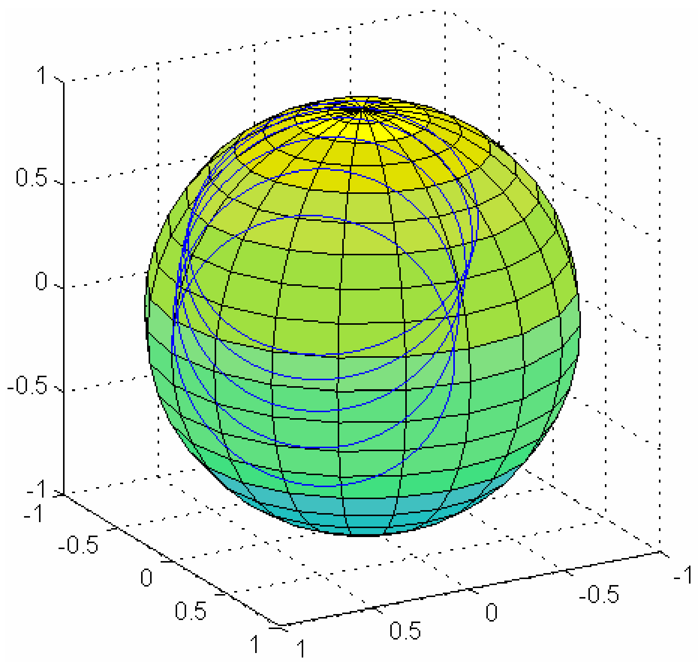

Fig.7. The trajectory of affix movement of the differential analyzer

The velocity of affix movement is constant.

8. CONTROLLING THE ACCELERATION

As it was mentioned above, controlling acceleration is very essential task. To have the ability of affix acceleration control, we should add one more equation to those mentioned above (10) [2]:

$$\ddot{x}_1^2 + \ddot{x}_2^2 + \ldots + \ddot{x}_n^2 = W^2. \tag{3}$$

The process of these equations solving is very complex and leads to enormous calculation resources usage, so in classical approaches [1] some redundancy and simplifications methods are used. No simplified models give satisfactory results for all holonomic automatic systems dynamics. Therefore, a new algorithm is proposed to control both velocity and acceleration of holonomic automatic system. This algorithm presupposes usage of argument control to control both velocity and acceleration of affix movement. In fact, classical approach that uses two blocks to control velocity and acceleration separately is replaced by new one, which uses only one block to calculate the argument of differential analyzer ω to control both velocity and acceleration of affix movement. The block-diagram for acceleration control is the same as on fig. 6, but demand velocity is time-varying. Velocity profile there is to be set up taking in mind the needed acceleration profile. Computer simulation results are presented on fig. 8, fig. 9, fig. 10.

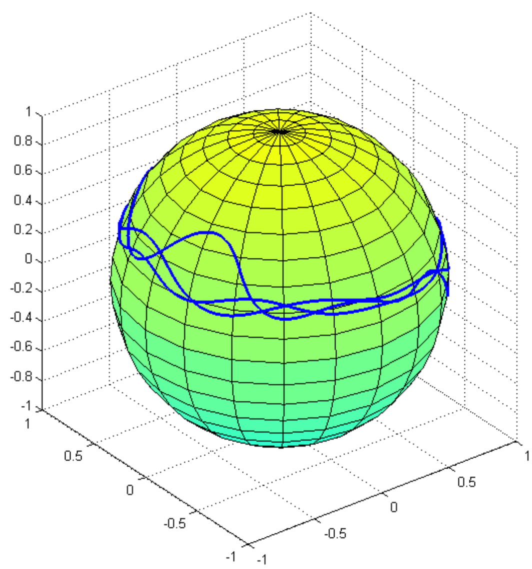

Fig. 8. Trajectory of affix movement

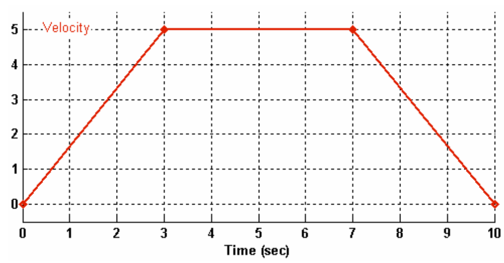

Fig. 9. Demanded velocity profile

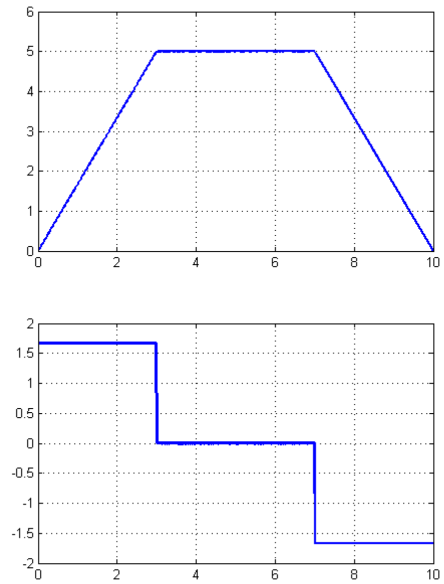

Fig. 10. Velocity and acceleration profiles of affix movement

A careful reader has noticed that velocity profile defines the acceleration profile. So, by assigning a definite velocity profile we can get affix movement dynamics with established velocity and acceleration. So, there is no need to use two sub-systems: one for acceleration control, another one for velocity stabilization, they both can be implemented in one sub-system which incorporates argument control according to a definite velocity, and, therefore, acceleration profiles.

9. CONCLUSION

1. The paper covers the methodology of differential analyzers synthesis for reproducing the curves lying on a quadric surface.
2. The computer simulation of differential analyzer war carried out.
3. When presenting the integrators' argument like $d\varphi=\omega dt$ it is possible to obtain a differential analyzer with argument control, which can be used to configure the dynamic properties of programmer, for example, to track the affix with pre-defined dynamic characteristics.
4. Argument control can be used to control both velocity and acceleration, as acceleration can be determined via velocity profile.

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